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Trajectory Method of Network Infrastructure Restoration

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Abstract—A method for restoration of a network distribution system damaged by an unfriendly impact is considered. The peculiarity of the method is the application of limit graphs to the solution of the restoration problem and the possibility of finding various optimization solutions taking into account the time and cost of work, the power shortage of sources, the number of switching operations of commutators, and others. The description of the method and restoration operations is given, taking into account the category of consumers. The assessment of the technical stability of the power distribution system is carried out on the example of a network fragment. The proposed approach can be used to formulate recommendations for improving the stability of infrastructure systems and to support the decision maker in real conditions of unfavorable factors.

Keywords: network infrastructure, engineering resilience, restoration

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1. INTRODUCTION

Engineering network systems are considered critical infrastructures, vital for the economy of any state, region or settlement. The elements of the network infrastructure (overhead and underground lines, pipelines, transformer and pumping stations, switchgear, etc.) are subject to internal and external adverse impacts (AI), which often cause accidents beyond the design requirements and lead to system degradation due to the destruction of numerous network elements and mass disconnection of consumers.

Of particular importance are the consequences of external climate impacts, the number and intensity of which have increased significantly in recent years due to climate change. The assumption of climate stationarity, on the basis of which the engineering systems were designed, is no longer obvious [1]. This means that the damage caused by natural hazards such as floods, hurricanes, earthquakes, etc., will only increase [2].

External impacts are often thought of as localized attacks. They can be divided into attacks with fixed and moving impact centers. In the first case, such as an earthquake, the intensity of the impact decreases with distance from the stationary center [3], while in the second case, the intensity decreases with distance from the trajectory of the moving center [4]. Thus, the impact area of a hurricane is defined by the band of its motion: its central coordinate, direction, length, and width. With increasing distance from the center, the intensity and consequences of the impact decrease.

Due to the spatial location of engineering networks, regardless of the type of attack, damage is always concentrated in a specific geographic area. Therefore, when modeling impacts on spatial networks, it is convenient to use two-dimensional lattices to reflect the geographic location of network elements [5].

Network infrastructures, being expensive long-term systems, need to be protected against AI. For this purpose, organizational (training of personnel and equipment) and technical measures (improvement of equipment reliability, upgrading, etc.) are implemented in the operational phase to prevent large-scale damage. To verify the level of readiness of systems against predicted AI, models of such impacts and recovery strategies will be developed.

The chosen strategy has a great impact on the resilience of an engineering system — the most important property characterizing the ability of an infrastructure system to ensure the established technical and economic performance of operation under the influence of destabilizing factors and to allow the restoration of serviceability in an acceptable time [6]. Closely related definitions of engineering resilience are given in [7].

To quantify engineering resilience, performance-based metrics and their corresponding curves of its change under AI are generally used, which are often referred to as resilience curves in works on resilience studies of infrastructure systems [3].

There are several approaches to the formation of an infrastructure recovery strategy, which take into account network properties, recovery order, allocated time, resources, etc. Thus, in [8], the components prioritized for restoration are determined using a multi-criteria optimization model for the resilience index and restoration time by mixed integer programming. In [9], an optimization model that also uses mixed integer programming is proposed to improve the resiliency of interdependent water, gas, and electric networks under AI.

In [3], three commonly used strategies are considered in the lattice framework: periphery recovery (PR), preferential recovery based on nodal weight (PRNW), and localized recovery (LR). The weight of a node is usually associated with the importance of the connected consumers (hospitals, factories, schools, etc.) that the node provides with the resource.

According to the PR strategy, the recovery starts with the selection of an isolated node with the highest weight neighboring to the functional node, i.e., the node receiving the resource, and the repair of the link connecting them. Since there can be more than one neighboring functional node in the lattice, the selection of lines for repair is random. After all isolated nodes of the same weight are connected at the current step, another isolated node with the highest weight neighboring the functional node in the lattice is selected at the next step and the line connecting them is repaired, etc. In the PRNW strategy, the recovery advantage is given to the links that connect primarily the isolated node with the highest weight to the functional node of the network and the links of this node to other significant nodes. The selection of damaged links incident to a functional node is also randomized. In LR strategy, recovery starts with selecting the root node and repairing its corrupted links. The links of the remaining nodes are restored in the order of the distance of these nodes from the root node, regardless of their weight. These strategies utilize a gradual step-by-step recovery of links and supply the previously isolated nodes incident to them.

The disadvantages of strategies that use optimization are the complexity of building a model with a large number of constraints and the duration of calculations. The disadvantages of the last three strategies includes: random selection of lines to be repaired, which creates a multiplicity of repair trajectories; and the possibility of connecting nodes with the highest weight at the end of the repair process (PR, LR). Considering only lines as damage creates a simplified picture of the impact.

The real picture of damage differs from the one described above. For example, even at high intensity, some overhead lines may not be damaged, e.g., if they are located in the wind direction. Part of the nodes (e.g. substations) may be destroyed due to insufficient resilience of the structure. Within the affected area there may be isolated "islands" of undamaged network elements, which should be taken into account in the restoration process, etc. Therefore, the real infrastructure after the impact appears as a mosaic of network elements that remain operational and damaged. The damaged elements themselves are part of the cross-sections that disrupt the reachability of nodes not affected by the attack to the resource sources.

Uninterrupted power supply after natural and man-made disasters cannot be ensured without prioritization of loads and resources [10] .Therefore, when developing the strategy, it is necessary to determine the composition of elements to be restored and the priority (order) of damage restoration, taking into account the priorities of consumers left without resources.

For example, the restoration of those elements that provide resources to the more important customers (first category customers) should be performed first. In other cases, restoration should be carried out in such a way that resources are provided to all consumers, but with limited supply volumes, etc. Depending on the priorities, the resource supply paths and the composition of recoverable elements are formed. It is clear that these paths may not repeat those that existed before the crisis.

The selected composition of repairable elements and the order of damage repair should meet certain criteria and take into account the presence of constraints in resources: financial, material (machinery) and human (repair crews). Most often, the cost and timing of work are used as criteria, then the other available resources in the optimization problem act as constraints.

Below is an approach to restoring infrastructure network elements (nodes and lines) based on a real picture of damage, which mitigates the shortcomings of the above strategies.

An electrical distribution network is considered as an infrastructure system. Such a network is usually characterized by an open loop structure in which each consumer receives a resource from a single source. At the same time, a single source can provide resources to several consumers.

In [11], the notion of a "limit" graph was introduced to study networks with these properties. In [12, 13] algorithms for constructing the complete set of limit graphs are developed, theorems justifying the correctness of the algorithms are proved, and the application of the method to the solution of distribution network reconfiguration problems is considered. In the presented paper, the results of these works are extended to the solution of the problem of network infrastructure restoration after an outage.

The restoration path method discussed below can be visualized as a sequence of steps:

- search for paths between disconnected consumers and resource sources,
- determination of minimal sets of damaged elements (nodes and links) along these paths,
- selecting among the obtained sets the ones that require minimum time or cost to restore the elements,
- assessment of engineering stability.

These steps are detailed in the example of Section 4. For deterministic stability assessment, an integral performance indicator is chosen, which well reflects the overall state of the system in different modes, including the period of recovery after an AI [14].

2. PROBLEM STATEMENT

The restoration process is considered on the distribution network model, which is a connected graph $G =< V, E >$, where V and E are finite sets of vertices (network nodes) and edges (network links), respectively. In turn, the set of nodes of the graph can be represented as $V =< S, P, U >$, where S, P, U are, respectively, subsets of nodes of resource generation (sources), consumption (loads), conversion (substations) and resource distribution (distributors). The consumption nodes are connected to the resource conversion nodes.

In the following we will assume that all links of the set E are switched and the switches located on them can be in two states: "closed," when the resource passes through the link, and "open," when the resource does not pass through it. According to the state of the switches, we can talk about the closed or open state of the edges.

Network operation modes depend on the state of switches, the control of which is used to change the configuration of the network and, as a rule, is carried out according to the criteria of minimizing power losses in the lines, equalization of loads in the lines, etc.

Thus, the resource allocation routes are defined by sets of switch states. Each set is characterized by a certain direction of flows in the network and the corresponding orientation of graph edges. Due to the use of different sets of states in mode control, the orientation of edges can be changed. For this reason, the paper uses terminology adopted for undirected graphs.

The question arises how the resurvival of links in distribution networks affects the sets of switch states, for example, under unloaded ("cold") and loaded ("hot") reserves. In unloaded reserve, each link contains switches in different states that are naturally included in the sets. In case of loaded reserve, the switches of the reserved links must be in the "closed" state. In this case, it is recommended to switch to an equivalent scheme where parallel links are combined into one with a common resource flow and a single switch.

In order to distinguish among the sets of states those that are inherent to the distribution network, we use the definition of graph "configuration" introduced in [13].

For any subset of edges $W \subseteq G$, we define the following operation:

1) remove the subset W from the graph G;

2) remove from the graph G all edges and vertices that do not lie on any path $s \to p$, where $s \in S$, $p \in P$.

Let us call the described operation the construction of a "allowed" subgraph.

A source vertex $s \in S$, can also be a deleted vertex if there is no path $s \to p$ for any $p \in P$.

Thus, we consider a subgraph G, containing paths $s \to p$, where $s \in S$, $p \in P$, that does not include any edges in the open state.

We will call the resulting allowed subgraph a configuration graph τ_r, if for every vertex $p \in P$ there exists a path $s \to p$ from at least one vertex $s \in S$.

We will also call the configuration graph a limiting graph $\tau_r^{\lim}$, if, after removing any edge $e \in E \setminus W$ from it, there is a vertex $p \in P$, for which there is no path $s \to p$, where $s \in S$.

In [10] it is shown that every limit graph $\tau_r^{\lim}$ has the following properties:

1) for a vertex from P there are no two paths leading from different vertices from S;

2) $\tau_r^{\lim}$ does not contain cycles, the directions of edges are determined uniquely by the paths $S \to P$.

Properties 1 and 2 are specific to distribution networks.

After the occurrence of AI, the set of vertices U splits into two subsets: the subset of vertices $\widetilde{U}$, disconnected from the sources and the subset of vertices remaining connected to the sources, hereafter referred to as the functional vertices $\overline{U}$.

At the same time, $\widetilde{U} = \widetilde{U}^* \cup \widetilde{U}^0$, where $\widetilde{U}^*$ is the subset of vertices that are physically damaged and $\widetilde{U}^0$ is the subset of vertices isolated from the sources by damaged vertices and edges.

The set of edges E will also divide into a subset of edges $\widetilde{E}$, disconnected from the sources and a subset of edges remaining connected to the sources, hereafter referred to as the functional edges of $\overline{E}$.

At the same time, $\widetilde{E} = \widetilde{E}^* \cup \widetilde{E}^0$, where $\widetilde{E}^*$ is the subset of edges that are physically damaged, $\widetilde{E}^0$ is the subset of edges isolated from the sources by damaged vertices and edges.

Isolation or damage of a node means disconnection from the resource of a consumer from the subset $\widetilde{P}$, $\widetilde{P} \subseteq P$, connected to this node.

Thus, as a result of AI we have the following subsets of network elements: physically damaged edges $\widetilde{E}^*$ and vertices $\widetilde{U}^*$, remaining functional edges $\overline{E}$ and vertices $\overline{U}$, healthy, isolated edges $\widetilde{E}^0$ and vertices $\widetilde{U}^0$.

After analyzing the network structure changed as a result of AI, the problem statement is presented in the following form. On the graph $G =< V, E >$, some of the vertices and edges of which are in a damaged state, due to which the vertex-consumers of the set $\widetilde{P}$ do not receive resources, it is required to find such paths $S \to \widetilde{P}$, that contain the minimum power sets of damaged vertices and edges, the restoration of which provides resources to the disconnected consumers of the set $\widetilde{P}$.

Under certain conditions, the specified minimum sets provide recovery in the least amount of time or cost. The vertices and edges of these sets have the highest recovery priority.

In [13] it is shown that the complete set of limit graphs corresponds to the complete set of consumer-source connections. Therefore, there exists a limit graph containing the required set of paths. In the same paper, methods for finding the complete set of limit graphs are investigated, i.e., methods of simple path combinations and the component method. Therefore, it is further assumed that the set of limit graphs $\tau_r^{\lim}$ is defined.

Next, consider a subset of the damaged vertices $\widetilde{U}^*$ and edges $\widetilde{E}^*$ and identify those that should be prioritized for repair to restore supply to consumers from $\widetilde{P}$.

3. LIMIT GRAPHS IN THE RECOVERY PROBLEM

As in the strategies discussed in the introduction, we assume that the restoration of a damaged network element (node or link) takes place after its repair and connection to a node of subset $\overline{U}$, i.e., to a node whose functionality has not been disrupted by AI.

Let $\tau_r^{\lim}$ be the complete set of limit graphs of graph G. Then those subsets of vertices $\widetilde{U}^*$ and edges $\widetilde{E}^*$, that belong to the limit graph $\widetilde{\tau}_{r0}^{\mathrm{im}}$, will have a higher recovery priority than the rest of the damaged elements.

On the set of vertices $\overline{U}$, consider a subset of boundary vertices $\overline{U}_s$.

A vertex $\overline{u}$ of a set $\overline{U}$, located on a path $s \to \widetilde{p}$, where $s \in S$, is called a boundary vertex for this path if all vertices and edges between $\overline{u}$ and $\widetilde{p}$ belong to sets $\widetilde{U}$ and $\widetilde{E}$, respectively, i.e., there is a boundary vertex that is the functional vertex closest to the disconnected consumer.

Construct a subgraph $\widetilde{G}$ of the network graph G, in which the vertices of the set $\overline{U}_s$, are considered as source vertices and the vertices of the set $\widetilde{P}$ as consumer vertices. We add to this subgraph the vertices of the set $\widetilde{U}$ and edges of the set $\widetilde{E}$. Then we remove all vertices and edges that do not lie on paths $s \to \widetilde{p}$, where $s \in \overline{U}_s,\ \widetilde{p} \in \widetilde{P}$.

We denote $\widetilde{\tau}_r^{\lim}$ the complete set of limit graphs for a graph $\widetilde{G}$ and recall that $\tau_r^{\lim}$ is the complete set of limit graphs for a graph G. When using the component method [13] the boundary functional vertices of the $\overline{U}_s$ set are determined simultaneously with the construction of the set of limit graphs $\widetilde{\tau}_r^{\lim}$.

The computational complexity of recovery options is greatly reduced when using limit graphs $\widetilde{\tau}_r^{\lim}$ instead of τ_r^{im}. This is due to considering the vertices of the set $\overline{U}_s$ as sources for consumers $\widetilde{p} \in \widetilde{P}$. As a consequence, the set $\widetilde{\tau}_r^{\lim}$ is defined on a significantly smaller number of vertices compared to the set $\tau_r^{\lim}$.

We denote by $\widetilde{\tau}_{r0}^{\lim}$ the graph belonging to the set $\widetilde{\tau}_r^{\lim}$ and containing the smallest number of corrupted elements: vertices from $\widetilde{U}^*$ and edges from $\widetilde{E}^*$.

Then the following statement is true

Statement 1. *Priority in recovery of damaged vertices and edges belong to vertices $\widetilde{u^*}$, $\widetilde{u^*} \in \widetilde{U^*}$ and edges $\widetilde{e}$, $\widetilde{e} \in \widetilde{E}$, that are included in $\widetilde{\tau}_{r0}^{\lim}$.*

This is not difficult to prove. The complete set of limit graphs $\tau_r^{\lim}$ corresponds to all admissible variants of joining vertices of P to vertices of S, including all admissible variants of joining vertices of $\widetilde{P}$ to vertices of S, since $\widetilde{P} \subseteq P$. In $\tau_r^{\lim}$, all paths $S \to \widetilde{P}$ pass through the vertices of $\overline{U}_s$ and hence the paths $\overline{U}_s \to \widetilde{P}$ pass through the vertices of the set $\widetilde{U}$ and the edges of the set $\widetilde{E}$, which are in $\widetilde{G}$. Then the subgraphs of limit graphs $\tau_r^{\lim}$, consisting of paths $\overline{U}_s \to \widetilde{P}$, form the complete set of limit graphs $\widetilde{\tau}_r^{\lim}$.

These graphs $\widetilde{\tau}_r^{\lim}$ correspond to all admissible variants of connecting vertices $\widetilde{P}$ to vertices $\overline{U}_s$, and hence correspond to all possible variants of restoring power supply to consumers $\widetilde{p}$, $\widetilde{p} \in \widetilde{P}$ from vertices $\overline{U}_s$ and, hence, from sources s, $s \in S$.

Clearly, if we restore the nodes $\widetilde{u^*}$, $\widetilde{u^*} \in \widetilde{U^*}$ and $\widetilde{e^*}$, $\widetilde{e^*} \in \widetilde{E^*}$, that are included in $\widetilde{\tau}_{r0}^{\lim}$, the restoration of consumer supply will occur with minimum lead time and cost (under the assumptions made above).

The limit graphs in $\widetilde{\tau}_r^{\lim}$ will be called recovery graphs hereafter.

4. APPLICATION OF THE METHOD TO THE RESTORATION OF A DAMAGED NETWORK

The recovery in the proposed method is carried out according to the following steps:

1) Selection of consumers of a given category.

2) Construction of a subgraph $\widetilde{G}$.

3) Construction of a set of limit graphs $\widetilde{\tau}_r^{\lim}$ (recovery graphs).

4) Selection of network elements — candidates for restoration and assessment of engineering stability of the network infrastructure.

Stages 1–4 are repeated for consumers of lower categories (if any).

Stages 1–3 are considered above, and therefore it is further assumed that the set of recovery graphs is constructed. Let us consider step 4 in more detail on the example of the network of Fig. 1a.

Figure 1a shows an example of a distribution network graph. "Bold" lines indicate the substation supply scheme adopted for normal operation (hereinafter referred to as "normal" scheme).

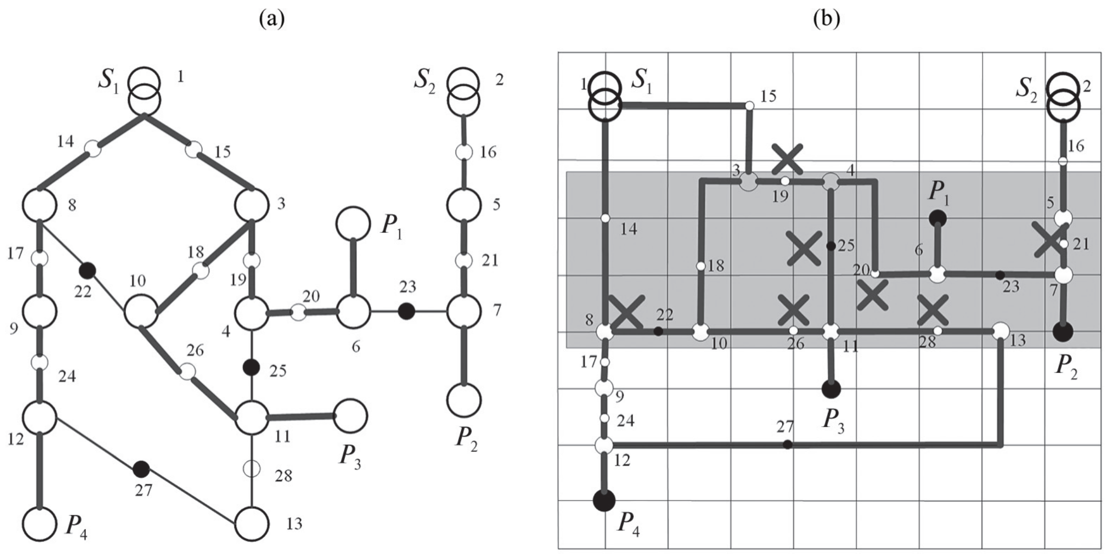

Fig. 1. Supply of consumers P_1–P_4 from sources S_1 and S_2: (a) graph of the network system; (b) location of the network system in the geographical area.

The network graph has two nodes — sources S_1 and S_2, which can be power centers, functional nodes 3–13, which correspond to the substations of the network, 15 network lines 14–28. Each substation has a load connected to it, which determines the current capacity of the substation. Four of them P_1–P_4 are shown in Fig. 1. "Bold" lines indicate the substation supply scheme adopted for normal operation. Small circles are edges, which correspond to sections of overhead lines and at the same time to switching devices. Large circles correspond to vertices, i.e. substations of the network. The "open" state of the switching devices in Fig. 1 is represented by a dark circle, the "closed" state by a white circle. Figure 1b shows the location of network elements on a two-dimensional lattice. The area of a negative event (e.g., a hurricane) is represented as a shaded band. The set of network system elements damaged as a result of this event is marked with crosses. The "normal" scheme of Fig. 1a corresponds to the limit graph $\tau_r^{\lim}$.

For the example of Fig. 1, we introduce the following assumptions that do not affect the proposed approach to the restoration problem:

• all substations before AI had the same capacity (further equal to 1 conventional units),

• the repair cost of any network element is the same,

• repairs are performed sequentially by one crew at the rate of one network element per unit of conditional time,

• the time for switching operations in switching devices can be neglected compared to the time for repairing a network element.

We denote by $\widetilde{\tau}_{r0}^{\lim}$ the graph containing the smallest number of vertices from $\widetilde{U}^*$ and $\widetilde{E}^*$. Under the above assumptions, restoration of damaged vertices and edges included in $\widetilde{\tau}_{r0}^{\lim}$, will result in restoration of supply to consumers in minimum time and at minimum cost.

For this reason, the graph $\widetilde{\tau}_{r0}^{\lim}$ is searchable and its vertices of set $\widetilde{U}^*$ and edges of set $\widetilde{E}^*$ have the highest recovery priority.

4.1. Selecting Network Elements — Candidates for Restoration

This stage includes selection of a set of network elements from the composition of damaged elements, optimal for restoration.

Let us consider step 4 on the example of the distribution network of Fig. 1. The sets of physically damaged nodes $\widetilde{U^*} = \{8\}$, edges $\widetilde{E}^* = \{19, 20, 21, 25, 26, 28\}$. The set of isolated nodal vertices $\widetilde{U^0} = \{4, 6, 7, 9, 11, 12, 13\}$ and edges $\widetilde{E^0} = \{17, 23, 24, 27\}$. The boundary vertices are $\overline{U}_s = \{1, 3, 5, 10\}$.

For the sake of clarity, we will limit ourselves to restoring the functionality of the four nodes supplying consumers $\widetilde{P_1}$–$\widetilde{P_4}$, which does not affect the generality of the approach. Let consumers $\widetilde{P_1}$, $\widetilde{P_2}$ among them belong to the first category and $\widetilde{P_3}$, $\widetilde{P_4}$ to the second category.

The power supply of these consumers is carried out according to [15]. The consumers of the first category (hazardous industries, hospitals, etc.) are supplied with power from at least two independent power sources through independent lines, the second category (clinics, schools, etc.) — from two independent sources through independent lines, the third category (residential buildings, etc.) are supplied with power from a single source. We will consider the resumption of supply to consumers of the first category as a priority task.

Restoration of supply to consumers of the first category. Under the assumptions made, the minimum recovery cost C is achieved in a graph $\widetilde{G}$ with minimal power of the set $\widetilde{U^*}\cup\widetilde{E^*}$.

As an example, consider two limit graphs for the first category consumers $\widetilde{P_1}$, $\widetilde{P_2}$.

In the notation of vertices and edges in Table 1 we omit tilde and underscore, otherwise they coincide with the notation of the sets containing them: functional vertices $\overline{U}$, damaged vertices $\widetilde{U^*}$ and edges $\widetilde{E^*}$, isolated vertices $\widetilde{U^0}$ and edges $\widetilde{E^0}$.

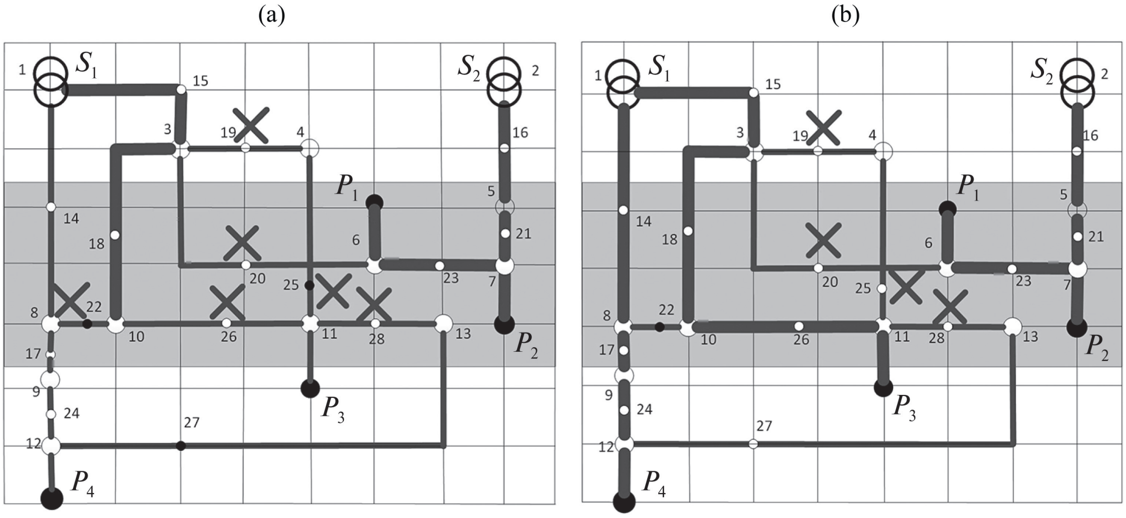

Fig. 2. Restoration of supply: (a) consumers of the first category $\widetilde{P_1}$, $\widetilde{P_2}$; (b) consumers of the second category $\widetilde{P_3}$, $\widetilde{P_4}$.

In Fig. 2a, the "thick" lines indicate the paths of energy supply to the functional nodes 3, 10, as well as the recovery graph $\widetilde{\tau}_{r0}^{\lim}$, constructed according to variant 1 of Table 1, which has a smaller number of "damaged" edges 21* compared to variant 2 (19*, 20*, 21*). "Serviceable" vertex 23^0 in the normal circuit corresponds to a switch that is in the "open" state. Variant 1 involves repairing the damaged line 21* as a first-order repair and placing switch 23^0 in the "closed" state.

Restoration of supply to lower category consumers. Let us assume that the restoration of supply to consumers of the first category is performed according to variant 1 of Table 1. As an example, consider two restoration graphs for the second category consumers $\widetilde{P_3}$, $\widetilde{P_4}$. After restoration of a part of the network for supplying the first category consumers, the damaged fragment of the network is changed and the choice of the elements to be restored is considered in the subgraph $\widetilde{G}$ corresponding to these changes.

To restore the supply $\widetilde{P_3}, \widetilde{P_4}$ in variant 1 of Table 2, it is required to repair lines $26^*, 28^*$ and perform switching in device 27^0.

Variant 2 of Table 2 is defined by the restoration graph, which includes the damaged substation 8^* and line 26^*.This variant is selected for the repair of the second line $\widetilde{\tau}_{r0}^{\lim}$. It is shown in Fig. 2b as a "thick" power distribution lines.

The preferred option for the same number of faults is selected by the decision maker, taking into account the number of unblocked (previously isolated) nodes, supply of consumers from different sources, number of switching operations [10] etc.

Table 1. Examples of supply recovery graphs of the first category consumers

No. of var.	$\overline{U}_s$ boundary nodes	Recovery graphs
1	5	$5, 21^*, 7^0, 23^0, 6^0, P_1; 5, 21^*, 7^0, P_2$
2	3, 5	$3, 19^*, 4^0, 20^*, 6^0, P_1; 5, 21^*, 7^0, P_2$

Table 2. Examples of supply recovery graphs of the second category consumers

No. of var.	$\overline{U}_s$ boundary nodes	Recovery graphs
1	10	$10, 26^*, 11^0, P_3; 10, 26^*, 11^0, 28^*, 13^0, 27^0, 12^0, P_4$
2	1, 10	$10, 26^*, 11^0, P_3; 1, 14^0, 8^*, 17^0, 9^0, 24^0, 12^0, P_4$

4.2. Engineering Stability Assessment

The assessment of engineering stability for the example network of Fig. 1 is performed after plotting its capacity variation under AI. To illustrate the approach to stability assessment, we will neglect the power loss in the lines and consider only the power change in the network nodes (substations). We also assume that the degradation is instantaneous.

A total of 8 nodes (4, 6, 7, 8, 9, 11, 12, 13) were damaged and isolated as a result of the adverse impact event, which is equivalent to a network performance loss of 8 c.u. As a result of AI, six lines 19, 20, 21, 25, 26, 28 and one substation 8 (Fig. 1a) were damaged. For this reason, only consumers remained connected to the only consumers of three substations 3, 5, 10 remained connected to the network, which have a total capacity of 3 c.u.

Consider the distribution of substation capacities for the selected restoration option. After line 21 is repaired and substation 7 is connected, the network capacity will increase to 4 c.u. After switching switch 23 to the "closed" state, substation 6 will be connected to the network of source S_2, and the network capacity will increase to 5 c.u. At the end of the second stage of repair, four of the previously disconnected substations 8, 9, 11, 12 will be connected to source S_1, and the network capacity will increase to 9 c.u. After switching switch 27 to the "closed" state, substation 13 will be connected to the network of source S_1, and the network capacity will increase to 10 c.u.

The recovery process for the repair options selected from Tables 1 and 2 can be visualized as a graph (Fig. 3).

On the graph, horizontal segments correspond to the network sections and substations to be restored, their designations are above these lines. Vertical segments indicate the change in performance due to the connection of substations to the functional part of the network, the numbers of substations are indicated to the right of these segments. The circles on the graph show switching nodes, the state of which is changed by switching from "open" to "closed."

The values t_h, t_e, t_d, t_s, t_f define the boundaries between the different phases of the process: the beginning of preparation for AI; absorption of impacts due to initial and subsequent failures due to internal properties of the system, e.g., redundancy; completion of failures and the beginning of the phase of damage assessment and development of recovery measures; transition to recovery; completion of system recovery. Due to the assumption of instantaneous degradation $t_e = t_d$.

In the recovery completion phase, the achieved productivity may not correspond to its pre-crisis value. The residual productivity is determined by its minimum level at the control interval $[t_0, t_c]$.

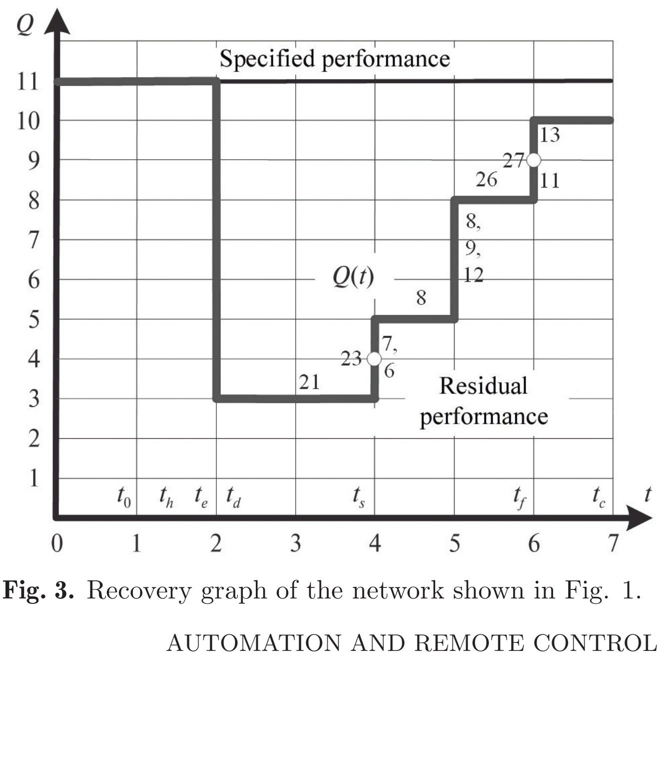

Fig. 3. Recovery graph of the network shown in Fig. 1.

The fraction of area bounded by the function $Q(t)$ on the reference interval $[t_0, t_c]$ of Fig. 3 relative to the target performance on that interval is used to evaluate the engineering stability of the system. This fraction, calculated for the graph of Fig. 3 and the reference measurement interval $[1, 7]$, is equal to 0.61.

The choice of characteristic moments of time that determine changes in the behavior of the system needs clarification. At present, there are no well-established formulations for defining the above time boundaries of phases and control interval. At the same time, at a large control interval, much longer than the time of completion of full recovery, the performance may be close to the target and the area ratio may tend to unity. This fact indicates the dependence of the obtained integral stability assessment on the choice of values t_0, t_c. The literature provides recommendations for their selection, usually application-dependent. Thus, to establish the value of t_c the following are used: expected, average, maximum recovery times; stakeholder considerations for a particular scenario; average time between unfavorable events.

The integral performance of the system J is evaluated as

$$J = \int_{t_0}^{t_c} Q(t)dt. \tag{1}$$

In [14] various indices and shapes of the stability curve are investigated. Among the widely used ones for deterministic evaluation of recovery performance, the normalized integral performance index (1) is applied, related to the control interval of evaluation performance $[t_0, t_c]$

$$\Phi = \frac{\int_{t_0}^{t_c} q(t)dt}{t_c - t_0}. \tag{2}$$

In (2) $q(t)$ denotes productivity normalized by its given value.

The recovery cost C is equal to $C = rT$, where r is the cost per unit time.

If the value of the indicator Φ does not meet the specified value, it is necessary to revise the options or change the restrictions on the number of repair teams, technical means, etc.

When implementing the described strategy, the configuration of the distribution network has changed compared to the supply scheme in the pre-crisis situation (Fig. 1a). Therefore, at the next stage of recovery, if financial and technical means are available, it is possible to reconfigure the network, raising its performance to 100% in the same way as described above, by changing the states of switches and repairing the remaining damaged network elements.

After selecting the restoration option, the values of line flows and busbar voltages should be checked to prevent them from exceeding the permissible values. For this purpose, methods of mathematical modeling of flow distributions in electrical networks are used.

5. CONCLUSION

An approach to restoring network infrastructure damaged as a result of adverse effects is proposed. A special feature of the approach is the application of the limit graph method, previously developed for solving network reconfiguration problems, to restoring a damaged network. The full set of limit graphs obtained using this method allows finding various optimization solutions for performing restoration, taking into account the timing and cost of work, the shortage of power sources, the capacity of lines, the number of switches, etc.

Time spent on calculation of limit graphs in the restoration problem is reduced due to the possibility of considering only the part of the network limited by the zone of unfavorable impact. Each solution should be verified by mathematical modeling of flow distribution processes.

Creating a system that is robust to extreme adverse impacts presupposes the availability of organizational mechanisms and technical means for its recovery. The proposed method can be used to assess the effectiveness of the organization's means of recovery by simulating probable adverse impacts. Based on the assessment of robustness, recommendations for improving measures to counteract unfavorable factors can be formed. At the same time, this method is aimed at supporting the decision maker in the real conditions of adverse events.

Not all the possibilities of applying the method of limit graphs to the restoration of a damaged network are considered in the article, in particular, the issues of restoration of damaged infrastructure with regard to line overloads, power shortages at consumers, the applicability of the method to interacting infrastructure networks, etc. need to be investigated.

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